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A globally convergent primal-dual predictor corrector interior point algorithm for linear programming has been developed.

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FINAL TECHNICAL REPORT

Grant Number: AFOSR-92-J0046

Principal Investigator: David F. Shanno

Grantee: Rutgers - The State University of New Jersey
Research Contract Administration
New Brunswick, New Jersey 08903

Research Title: Numerical Methods for Linear and
Nonlinear Optimization

Dates: 11/15/91 - 4/15/95

Summary

Research during the period covered by the grant continued to develop interior point methods for large scale linear programming problems. Specific accomplishments included a decomposition method for a large crew-scheduling problem which combined interior point and simplex methods. The method solved a previously intractable problem in 4 minutes on a CRAY Y-MP. More recently, an improved version of this algorithm has successfully solved the problem on a single processor Silicon Graphic Power Challenge machine. A globally convergent version of the algorithm was developed, and shown to be computationally efficient. The linear programming work culminated in an invited state-of-the-art paper.

Beyond linear programming, the major focus of the research was on nonlinear programming. Here extensive study was made of modified logarithmic barrier functions for nonlinear programming. This culminated in a penalty-barrier algorithm which has successfully solved several test sets of nonlinear programming problems, a major advance in the status of interior point methods. Further, specific implementations were designed and tested for separable nonlinear and quadratic programming problems, and for linear and nonlinear complementarity problems, which are generalizations of nonlinear programming problems. In all cases, significant improvements were made over the computational

performance of known methods.

Detailed Description

During the period covered by the grant, this research was directed into four specific areas. All were concerned with extending the previous work of the principal investigator in interior point methods. The four areas studied were interior point methods for linear programming, extensions of linear programming methods to general quadratic and separable nonlinear programming, modified barrier methods for general nonlinear programming, and barrier methods for nonlinear complemantarity problems. The report will discuss each in turn.

One major research problem for linear programming concerned a hybrid simplex-interior-point decomposition algorithm for a crew scheduling model developed by American Airlines. The problem had 827 constraints, but approximately 6 million variables. The algorithm successfully solved the problem on a Cray Y-MP in about 4 minutes. This work was instrumental in leading to the implementation of the primal-dual predictor-corrector method in the CPLEX linear optimization package. Continued development in this framework of both interior point methods and hybrid methods have led to significant improvements in computer time for this and similar problems. This work is documented in Operations Research in the paper [5] listed in the Publications.

A second major topic of study in linear programming was devising and testing a version of the primal-dual predictor-corrector code which could be proved to be globally convergent. A globally convergent algorithm was devised, programmed, and tested, and shown to be computationally efficient. This work is documented in Mathematical Programming in reference [10].

The work on linear programming culminated in two comprehensive invited papers, one on the state-of-the-art in interior point methods and the other in all computational methods for linear programming. These are documented in [8] and [11].

A simple and natural extension of interior point methods for linear programming is to interior point methods for separable quadratic and linearly constrained nonlinear programming problems. The separable nature of the problems assures that the matrix of second partial derivatives is diagonal, which is precisely the case which occurs when introducing nonlinearity to linear programming by incorporating the nonregativity constraints in a logarithmic barrier term. In this case, the application of the methods developed for linear programming is straightforward. The methods have been implemented and tested comparatively against MINOS, a very successful quadratic and nonlinear programming code. On the tested problems, the interior point code has proved to be a substantial improvement. A version of the quadratic programming algorithm has been implemented and tested in CPLEX, and will be included with the next release. The work has been documented in [4] and [6].

For quadratic problems with large nonsparce Hessians, a conjugate projected gradient variant of an interior point code has been programmed and tested and documented in [7]. Further testing of this algorithm awaits a better set of test problems.

The next target of the research was to concentrate on general nonlinear programming problems, where the constraints are allowed to be nonlinear and no assumption is made as to separability of the objective function. This is in fact the problem originally studied by Fiacco and McCormick which led to the initial work on logarithmic barrier methods. Over time, serious numerical problems with the initial approach of Fiacco and McCormick led to the methods being replaced by more robust algorithms. Recently, motivated by work of Roman Polyak, modified barrier methods have been studied as a way of overcoming these difficulties with the classic logarithmic barrier method. Marc Breitfeld, a Ph.D. student at Rutgers, studied these methods extensively under the supervision of the principal investigator as the subject of his Ph.D. research. This work is documented in [9], [12], [14], and [15].

The results of this work are that a much more stable and efficient interior point method for the solution of general nonlinear programming problems has been devised. The method does not require an initial feasible point, does not become ill-conditioned as the optimum is approached, and does not require complex line search techniques. It is relatively insensitive to the choice of the barrier parameter, and includes equality constraints successfully through the addition of an augmented Lagrangian term to the barrier function. All of these additions make the algorithm far more stable and efficient than classical logarithmic barrier methods for nonlinear programming.

The greatest sensitivity of the new algorithm is to the step-by-step estimation of Lagrange multipliers. When the algorithm has proved inefficient or unstable, this has always been the underlying cause. Research is continuing to determine better ways to estimate Lagrange multipliers to overcome this problem.

The last topic considered under the grant was a generalization of nonlinear programming, the nonlinear complementarily problem. This includes nonlinear programming, n-person game theory, and some variational inequalities, among other applications. The work done under this grant was very preliminary, but promising, and is documented in [13]. Current research has carried the development of this preliminary research much farther, and fully developed algorithm is now solving an extensive test set more efficiently and stably than any previously known algorithm.

Publications 11/15/91 - 4/15/95

[1] "On implementing Mehrotra's predictor - corrector interior point method for linear programming", SIAM JOURNAL ON OPTIMIZATION 2, 1992, 435-449 (with I.J. Lustig and R.E. Marsten).

[2] "The interaction of algorithms and architectures for interior point methods", ADVANCES IN OPTIMIZATION AND PARALLEL COMPUTING, P. Pardolos, ed., Elsevier Science Publishers B.V., Amsterdam, The Netherlands, 1992, 190-205 (with I.J. Lustig and R.E. Marsten).

[3] "The interior point method for linear programming", IEEE SOFTWARE 9, July 1992, 61-68 (with G. Asfalk, I.J. Lustig, and

R.E. Marsten).

[4] "Separable quadratic programming via a primal-dual interior point method and its use in a sequential procedure," ORSA JOURNAL ON COMPUTING 5, 1993, 182-191 (with T.J. Carpenter, I.J. Lustig, and J.M. Mulvey).

[5] "Very large-scale linear programming: A case study in combining interior point and simplex methods," OPERATIONS RESEARCH 40, 1992, 885-897 (with R.E. Bixby, J.W. Gregory, I.J. Lustig and R.E. Marsten).

[6] "Higher order predictor-corrector interior point methods with application to quadratic objectives," SIAM JOURNAL ON OPTIMIZATION 3, 1993, 696-725 (with T.J. Carpenter, I.J. Lustig and J.M. Mulvey).

[7] "An interior point method for quadratic programs based on conjugate projected gradients," COMPUTATIONAL OPTIMIZATION & APPLICATIONS, 2, 1993, 5-28 (with T.J. Carpenter).

[8] "Interior point methods for linear programming; computational state of the art," ORSA JOURNAL ON COMPUTING 6, 1994, 1-14 (with I.J. Lustig and R.E. Marsten).

[9] "Preliminary computational experience with modified log-barrier functions for large-scale nonlinear programming," LARGE SCALE OPTIMIZATION: STATE OF THE ART, W.W. Hager, D.W. Hearn and P.M. Pardolos, Editors, Kluwer Academic Publishers, 1994, 45-67, (with M.G. Breitfeld).

[10] "Computational experience with a globally convergent primal-dual predictor-corrector algorithm for linear programming," MATH PROGRAMMING, 66, 1994, 123-135, (with I.J. Lustig and R.E. Marsten).

[11] "Computational methods for linear programming," ALGORITHMS FOR CONTINUOUS OPTIMIZATION, E. Spedicato, ed. Kluwer Academic Publishers, 1994, 383-414.

[12] "A globally convergent penalty-barrier algorithm for nonlinear programming" OPERATIONS RESEARCH PROCEEDING 1994, V. Derig, A. Bachem, and A. Drexl, eds., Springer-Verlag, 1995, 22-27 (with M.G. Breitfeld).

Accepted, not yet published 11/15/91 - 4/15/95

[13] "Computational experience with logarithmic barrier methods for linear and nonlinear comlementarity problems," OPERATIONS RESEARCH: METHODS, MODELS AND APPLICATIONS, J.E. Aronson & S. Zionts, eds., Quorum Books (in press) RRR 18-93, RUTCOR, Rutgers University, New Brunswick, NJ, October, 1993.

[14] "Computational experience with modified log-barrier methods for nonlinear programming," ANNALS OF OPERATIONS RESEARCH (in press) RRR 17-93, RUTCOR, Rutgers University, New Brunswick, NJ, October, 1993 (with M.G. Breitfeld).

Submitted, not yet accepted 11/15/91 - 4/15/95

[15] "A globally convergent penalty-barrier algorithm for nonlinear programming and its computational performance," RRR 12-94, RUTCOR, Rutgers University, New Brunswick, NJ, April 1994 (with M.G. Breitfeld).

Invited talks 11/15/91 - 4/15/95

November 1992	ORSA San Francisco (1-1/2 hour tutorial)
December 1992	Technical University of Berlin
January 1993	APMOD 93 Budapest (3 talks, 1 plenary)
February 1993	Large Scale Optimization, U. of Florida, Gainesville
September 1993	NATO Advanced Study Institute, Il Ciocco, Italy (4 hours plenary)
November 1993	ORSA, Phoenix
January 1994	ORSA Computer Science Tech. Sec. Conference Williamsburg, VA (1-1/2 hour tutorial)
August 1994	German OR Society, Berlin (Plenary)
November 1994	Conf. on Engineering Design, Lovain, Belgium (Plenary)
November 1994	Conf. on Transportation, Scheveningen, the Netherlands (Plenary)
January 1995	Math Research Institute, Oberwolfach, Germany (1 hour tutorial)
January 1995	Technical University of Munich
February 1995	Georgia Institute of Technology
April 1995	Informs, Los Angeles